

Examples of Inner products

- The dot product $\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + \dots + u_nv_n$ is the **Euclidean inner product** or **standard inner product** on R^n . [R^n with the Euclidean inner product is called **Euclidean n-space**.]
- If $\mathbf{u} = (u_1, u_2, \dots, u_n)$ and $\mathbf{v} = (v_1, v_2, \dots, v_n)$ are vectors in R^n and w_1, w_2, \dots, w_n are positive real numbers, then the formula $\langle \mathbf{u}, \mathbf{v} \rangle = w_1u_1v_1 + w_2u_2v_2 + \dots + w_nu_nv_n$ is called the **weighted Euclidean inner product with weights w_1, w_2, \dots, w_n** .
- On M_{nn} , the set of $n \times n$ matrices: If $\mathbf{u} = U$ and $\mathbf{v} = V$ are matrices in the vector space M_{nn} , then the formula $\langle \mathbf{u}, \mathbf{v} \rangle = \text{tr}(U^T V)$ is the **standard inner product** on M_{nn} .
- If $\mathbf{p} = a_0 + a_1x + \dots + a_nx^n$ and $\mathbf{q} = b_0 + b_1x + \dots + b_nx^n$ are polynomials in P_n , then the **standard inner product** on P_n is $\langle \mathbf{p}, \mathbf{q} \rangle = a_0b_0 + a_1b_1 + \dots + a_nb_n$. (Note the similarity in form to the dot product.)
- If $\mathbf{p} = a_0 + a_1x + \dots + a_nx^n$ and $\mathbf{q} = b_0 + b_1x + \dots + b_nx^n$ are polynomials in P_n and if x_0, x_1, \dots, x_n are distinct real numbers, then the formula $\langle \mathbf{p}, \mathbf{q} \rangle = p(x_0)q(x_0) + p(x_1)q(x_1) + \dots + p(x_n)q(x_n)$ is the **evaluation inner product** at x_0, x_1, \dots, x_n .
- If $\mathbf{f} = f(x)$ and $\mathbf{g} = g(x)$ are two functions in $C[a, b]$, then $\langle \mathbf{f}, \mathbf{g} \rangle = \int_a^b f(x)g(x)dx$ defines an **inner product** on $C[a, b]$.
- If \mathbf{u} and \mathbf{v} are vectors in R^n expressed in column form, A is an invertible $n \times n$ matrix, and $\mathbf{u} \cdot \mathbf{v}$ is the Euclidean inner product on R^n , then the formula $\langle \mathbf{u}, \mathbf{v} \rangle = A\mathbf{u} \cdot A\mathbf{v}$ is the **inner product on R^n generated by A** . This is an example of a **matrix inner product**.

#16 A sequence of sample points is given. Use the evaluation inner product on P_3 at those sample points to find $\langle \mathbf{p}, \mathbf{q} \rangle$ for the polynomials $\mathbf{p} = x + x^3$ and $\mathbf{q} = 1 + x^2$.
 $x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 2$

#8 Use the inner product on R^2 generated by the matrix A to find $\langle \mathbf{u}, \mathbf{v} \rangle$ for the vectors

$\mathbf{u} = (0, -3)$ and $\mathbf{v} = (6, 2)$.

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$$

Example: A weighted Euclidean inner product
